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Effective Business Management in Uncertain Business Environment Using Stochastic Queuing System with Encouraged Arrivals and Impatient Customers

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Abstract

In competitive contemporary business environment, organizations are facing various challenges. Uncertain customer behavior and high competition are few of many challenges. In order to stay ahead organizations introduce various offers and discounts to attract customers. These offers and discounts encourage customers to visit the particular organization. These encouraged arrivals result in heavy rush at times. Due to this customers have to wait longer in queues before they can get the service. Long waiting times, results in customer impatience and a customer may decide to abandon the facility without completion of service, termed as reneging. Reneging is a loss to business. Hence, there is need to design a strategy in advance to reduce loss of business and smooth administration. A better strategy can be designed if performance of the particular system can be measured with some probability. An M/M/1/N queue is developed to analyze the system functioning under above mentioned challenges. The model is solved iteratively in steady state. Necessary measures of performance are derived. Economic analysis is also performed by introduction of cost-model. The results are presented through suitable charts and tables. Numerical illustration and sensitivity analysis of the model is also performed.

Keywords: Customer Impatience Encouraged Arrivals, Queuing Theory, Reneging, Stochastic Models.

1. Introduction and Literature Survey

Business environment now days is highly challenging due to factors like uncertainty, globalization, and competition. Engaging new customers and retaining existing customers requires a full proof strategy. In order to engage new customers firms often release offers and discounts. Let it be online stores or offline stores, every firm comes up with discounts and offers every now and then. These offers and discounts attract customers to visit the stores or online web portals. These attracted customers are termed as *encouraged arrivals* as discussed by (Som and Seth, 2017), (Jain, *et al.*, 2014) introduced the concept of *reverse balking*. They mentioned that an arriving customer gets attracted towards a system by looking in to large customer base. Whereas reverse balking deals with probability of joining and not joining the system, encouraged arrivals deal with percentage increase in customers due to discounts and offers. The phenomenon of

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encouraged arrivals can also be understood as contrary to discouraged arrivals discussed by (Kumar et. al., 2014). In their work they mentioned that discouraged arrivals occur in accordance to Poisson process defined by parameter $\frac{\lambda}{n+1}$. They mentioned that customers are discouraged to join once they look in to large system size. A detailed literature can be cited in work of (Kumar et. al, 2014) about discouraged arrivals. Due to encouraged arrivals, system experiences heavy load of customers. This puts service under pressure and system results in longer queues. Longer waiting times result in customer impatience and customers start to abandon the system without completing their service. This is termed as *customer impatience or reneging*. Notion of customer reneging is discussed pioneer work of (Haight, 1959) and (Ancker and Gafarian, 1963). Reneging is a loss to business, goodwill of the company, and revenue. A number of papers emerged on impatient customers since the inception of concept. (Wang *et al.*, 2010) extensively reviewed customer impatience. (Kapodistria, 2011) studied a M/M/1/N queue with customer reneging and considered abandonment of customers simultaneously. Further (Ammar *et al.*, 2012) studied a queue with discouraged arrivals and reneging using matrix method.

We propose that the arrivals may get attracted by looking in to discounts and offers offered by companies as a marketing policy which often results in heavy rush and gives rise to customer impatience. Owing to this practically valid aspect we develop a stochastic queuing system with customer impatience operating through single channel. This paper is an extension of the work done by (Som and Seth, 2017) by introducing the concept of customer impatience. The paper is arranged as per following details. 2nd Section deals with stochastic model formulation while 3rd section presents steady-state solution of the model. Measures of performance are derived in 4th section. Numerical illustration is presented in 5th section. Cost-profit analysis of the model is presented in 6th section. Conclusion of the paper is given in 7th section.

2. Mathematical Model Formulation

A single-server Markovian queuing model is formulated under following assumptions:

(i) The arrivals occur one by one in accordance to Poisson process with parameter $\lambda(1+\eta)$, where 'η' represents the percentage change in number of customers calculated from past or observed data. For instance, if in past an organization offered discounts and the percentage change in number of customers was observed + 25% or 150% then $\eta = 0.25$ or $\eta = 1.5$ respectively.

(ii) Customers are served with parameter μ exponentially.

(iii) First In First Out queue discipline is followed.

(iv) There is a single server through which the service is provided.

(v) Maximum permissible customers in the system are defined by N.

(vi) Reneging times are exponentially distributed with parameter ¾

Equations governing the model are given by:

$$\frac{d}{dt}P_0(t) = -\lambda(1+\eta)P_0(t) + \mu P_1(t) \quad (1)$$

$$\frac{d}{dt}P_n(t) = \lambda(1+\eta)P_{n-1}(t) + \{-\lambda(1+\eta) - \mu - (n-1)\xi\}P_n(t) + (\mu + n\xi)P_{n+1}(t) \quad (2)$$

$$\frac{d}{dt}P_N(t) = \lambda(1+\eta)P_{N-1}(t) - \{\mu + (N-1)\xi\}P_N(t) \quad (3)$$

As $t \to \infty, P_n(t) = P_n$ and therefore, $\frac{d}{dt}P_n(t) = 0 \; as \; t \to \infty$ and hence, equations (1) – (3) become;

$$0 = -\lambda(1+\eta)P_0 + \mu P_1 \quad (4)$$

$$0 = \lambda(1+\eta)P_{n-1} + \{-\lambda(1+\eta) - \mu - (n-1)\xi\}P_n + (\mu + n\xi)P_{n+1} \quad (5)$$

$$0 = \lambda(1+\eta)P_{N-1} - \{\mu + (N-1)\xi\}P_N \quad (6)$$

3. Steady-State Solution

On solving (4) - (6) iteratively we get;

$$P_n = Pr\{'n' customers\ in\ the\ system\} = \prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu + i\xi}P_0,\ 1 \le n \le N-1 \quad (7)$$

And the probability that system is full is given b··

$$P_N = Pr\{sytem\ is\ full\} = \prod_{i=0}^{N-1}\frac{\lambda(1+\eta)}{\mu + i\xi}P_($$

$$L_s = \sum_{n=0}^{N} nP_n$$

$$L_s = \sum_{n=0}^{N} n \times \left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu + i\xi}P_0\right\} \quad (10)$$

Using normality we have, $\sum_{n=0}^{N} P_n = 1$

$$P_0 = Pr\{system\ is\ empty\} = \left\{1 + \sum_{n=1}^{N}\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu + i\xi}\right\}^{-1} \quad (9)$$

4. Measures of Performance

1. Average Size of the System (L_s): It represents average number of customers in the system in stable state, which includes customers waiting for their service as well as those getting the service.

2. Average queue length (L_q): It refers to the average number of customers in the stable state, waiting in the queue for their service excluding those getting the service.

$$L_q = \sum_{n=0}^{N} (n-1)P_n$$

$$L_q = \sum_{n=0}^{N} (n-1)\left\{\prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0\right\} \qquad (11)$$

3. Average rate of reneging (R_r): It signifies the average number of customers per unit time who get impatient while waiting in the queue for time longer than their threshold limit and leaves the queue without getting service.

$$R_r = \sum_{n=1}^{N} (n-1)\xi P_n$$

$$R_r = \sum_{n=1}^{N} (n-1)\xi\left\{\prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0\right\} \qquad (12)$$

5. Numerical Interpretations

In this section we present numerical illustration of the above model.

Table 1: Variation in $L_s, L_q,$ *and* R_r with respect to λ

We take, $N = 10, \mu = 3, \xi = 0.2, \eta = 0.5$

Average rate of arrival(λ)	Average System Size(Ls)	Average Queue Length(L_q)	Average rate of Reneging (R_r)
2	3.093301	2.257036	0.451407
2.2	3.745812	2.864095	0.572819
2.4	4.412368	3.495343	0.699069
2.6	5.057981	4.114786	0.822957
2.8	5.655053	4.693244	0.938649
3	6.187427	5.212805	1.042561
3.2	6.649813	5.66658	1.133316
3.4	7.044665	6.055734	1.211147

3.6	7.37872	6.386048	1.27721
3.8	7.660334	6.665213	1.333043
4	7.897868	6.901141	1.380228
4.2	8.098879	7.101094	1.420219
4.4	8.269836	7.271349	1.45427
4.6	8.416106	7.41715	1.48343
4.8	8.542066	7.542794	1.508559
5	8.651257	7.651769	1.530354
5.2	8.74653	7.746894	1.549379

Source: simulated data

We can observe that with increase in arrival rate the expected system size increases and so as expected length of queue, while increase in rate of reneging means that lot of customers abandon the facility without completion of service. Following graph explains the phenomenon. Similarly, the numerical results are obtained by varying service rate.

Table 2: Variation in L_s, L_q *and* R_r with respect to μ

We take, $N = 10, \lambda = 3, \xi = 0.2, \eta = 0.5$

Average rate of service(μ)	**Average System Size(Ls)**	**Average Queue Length(L_q)**	**Average rate of Reneging(R_r)**
3	6.187427	5.212805	1.042561
3.1	5.998352	5.02838	1.005676
3.2	5.80758	4.842783	0.968557
3.3	5.615999	4.656907	0.931381
3.4	5.424503	4.471641	0.894328
3.5	5.233968	4.287852	0.85757
3.6	5.045235	4.106363	0.821273
3.7	4.859092	3.927938	0.785588
3.8	4.676261	3.75327	0.750654
3.9	4.497386	3.582968	0.716594
4	4.323025	3.417555	0.683511
4.1	4.15365	3.257462	0.651492
4.2	3.989641	3.103028	0.620606
4.3	3.831294	2.954508	0.590902
4.4	3.678821	2.812072	0.562414
4.5	3.532357	2.675814	0.535163
4.6	3.391968	2.545763	0.509153

Source: simulated data

We can observe that as the service rate increases, average size of system decreases and so as expected length of queue, while decrease in rate of reneging it means that large number of customers choose to wait rather than leaving the facility. Following graph explains the phenomenon.

6. Economic Analysis (Cost-Profit Analysis)

Cost-profit analysis of the model discussed by developing the functions of Total Expected Cost (TEC), Total Expected Revenue (TER) and Total Expected Profit (TEP).

Total expected cost of the system (TEC) is given by:

$$TEC = C_s\mu + C_h \sum_{n=0}^{N} n \left\{ \prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0 \right\} + C_r \sum_{n=1}^{N} (n-1)\xi \left\{ \prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0 \right\} + C_L \lambda \prod_{i=0}^{N-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0$$

Total expected revenue (TER) of the system is given by:

$$TER = R \times \mu \times (1 - P_0)$$

$$TER = R \times \mu \times \left[1 - \left\{ 1 + \sum_{n=1}^{N} \prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} \right\}^{-1} \right]$$

Total expected profit (TEP) of the system is given by;

$$TEP = (TER - TEC)$$

$$TEP = R \times \mu \times \left[1 - \left\{ 1 + \sum_{n=1}^{N} \prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} \right\}^{-1} \right] - C_s\mu + C_h \sum_{n=0}^{N} n \left\{ \prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0 \right\} + C_r \sum_{n=1}^{N} (n-1)\xi \left\{ \prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0 \right\} + C_L \lambda \prod_{i=0}^{N-1} \frac{\lambda(1+\eta)}{\mu + i\xi} P_0$$

Where, C_s= Cost /service /unit time, C_h= holding cost /unit /unit time, C_r= Cost related to each reneged /unit time, C_L= Cost related to each lost unit /unit time, R= Revenue earned / unit /unit time

The cost model formulated above if translated in MS EXCEL and sensitivity analysis is performed for varying rates of arrival and services.

Table 3: Variation in TEC, TER and TEP with respect to λ

We take, $N = 10, \mu = 3, \xi = 0.2,\ \eta = 0.5, C_s = 10, C_L = 15, C_h = 2, C_r = 2, R = 200$

Average arrival rate(λ)	TEC	TER	TEP
2	37.48741	501.7586	464.2712
2.2	39.45758	529.0299	489.5723
2.4	41.72146	550.2146	508.4931
2.6	44.23647	565.9167	521.6803
2.8	46.94663	577.0858	530.1392
3	49.7957	584.7734	534.9777
3.2	52.73611	589.9398	537.2037
3.4	55.73221	593.359	537.6268
3.6	58.7596	595.6032	536.8436
3.8	61.8027	597.0724	535.2697
4	64.85211	598.0359	533.1838
4.2	67.90247	598.6706	530.7682
4.4	70.95093	599.0916	528.1407
4.6	73.99611	599.3731	525.377
4.8	77.03751	599.5631	522.5255
5	80.07506	599.6924	519.6173
5.2	83.10896	599.7813	516.6724

Source: simulated data

The table shows that the profit increases with increment in average rate of arrival, reaches a maximum value at certain level and starts falling down. This is because of the fact that service rate being fixed, after certain level with increasing load on service, cost increases rapidly than revenue owing to longer queues and increasing reneging.

Table 4: Variation in TEC, TER and TEP with respect to μ

We take, $N = 10, \lambda = 3, \xi = 0.2, \eta = 0.5, C_S = 10, C_L = 15, C_h = 2, C_r = 2, R = 200$

Average service rate(μ)	TEC	TER	TEP
3	49.7957	584.7734	534.9777
3.1	49.88216	601.3826	551.5005
3.2	49.99321	617.4699	567.4767
3.3	50.1309	633.0008	582.8699
3.4	50.29708	647.946	597.6489
3.5	50.49332	662.2811	611.7878
3.6	50.72089	675.9879	625.267
3.7	50.98078	689.0541	638.0733
3.8	51.27361	701.4735	650.1999
3.9	51.59973	713.2458	661.646
4	51.95916	724.376	672.4168
4.1	52.35165	734.8741	682.5225
4.2	52.7767	744.7548	691.9781
4.3	53.23357	754.0362	700.8027
4.4	53.72135	762.7396	709.0182
4.5	54.23898	770.8885	716.6496
4.6	54.78529	778.5085	723.7232

Source: simulated data

The table shows that the revenue goes high and firms profit keeps on increasing with an improving rate of service due to reduced rate of reneging.

Table 5: Variation in TEC, TER and TEP with respect to ξ

We take, $N = 10, \lambda = 2, \mu = 3, \eta = 0.5, C_S = 10, C_L = 15, C_h = 2, C_r = 2, R = 200$

Reneging(ξ)	*TEC*	*TER*	*TEP*
0.1	39.2264	520.858	481.6316
0.15	38.21441	510.6823	472.4679
0.2	37.48741	501.7586	464.2712
0.25	36.95172	493.8918	456.9401
0.3	36.54641	486.9021	450.3557
0.35	36.23188	480.6386	444.4067
0.4	35.98206	474.9794	438.9973
0.45	35.77948	469.8272	434.0477
0.5	35.61218	465.1046	429.4925
0.55	35.4718	460.7496	425.2778
0.6	35.35237	456.7122	421.3598
0.65	35.24954	452.9514	417.7019
0.7	35.16007	449.4339	414.2738
0.75	35.08151	446.1316	411.0501
0.8	35.01198	443.0212	408.0093
0.85	34.95001	440.083	405.133
0.9	34.89442	437.3001	402.4057

Source: simulated data

The table shows that with increase in reneging rate the total expected cost drops slightly while TER and as a result TEP decreases as increasing rate of reneging means that lot of customers abandon the facility without completion of service which is loss to the business and revenue.

7. Conclusions and Future Scope

The results of the paper are of immense use for any organization encountering the phenomenon of encouraged customers and load on service. By knowing the measures of performance in advance the overall performance of the system can be measured and an effective strategy can be planned for smooth functioning. The margin of error in given times is so low, that a slight compromise with the strategy may result in heavy loss to the business.

By adopting and implementing this model the economic analysis of the facility can be measured and financial aspect of the business can also be observed.

Further optimization of service rate and system size can be achieved while the system can be studied in transient state. The system can also be studied for heterogeneous service. A multi-server model can also be developed. And the system can also be studied with infinite capacity.

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